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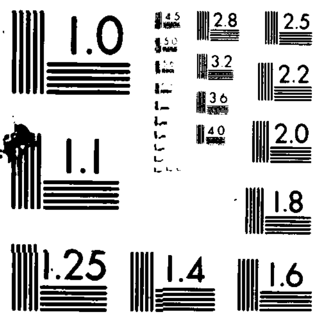

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Technical Report 742

DYNAMIC EQUATIONS FOR INITIALIZATION OF THE VERTICAL LAUNCH ASROC AUTOPILOT

DH Lackowski

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This report defines the reference coordinate frames and provides the dynamic equations required to support development of autopilot initialization algorithms in the fire control computer for the Vertical Launch ASROC system.

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CONTENTS

INTRODUCTION

This report defines the reference coordinate frames and provides the dynamic equations required to support development of autopilot initialization algorithms in the fire control computer for the Vertical Launch ASROC system. The report presents a mathematical definition of the dynamic data required by the missile autopilot for prelaunch initialization. These include:

(1) Attitude of the command reference frame relative to the autopilot reference frame

(2) Velocity of the launch point reference relative to the moving air mass

NOTATION FOR VECTORS AND ORTHOGONAL TRANSFORMATIONS

We understand that $\bar{r}$ represents a vector quantity without reference to any particular coordinate frame. If we wish to specify the vector $\bar{r}$ expressed in terms of $x_i y_i z_i$ coordinates, we use $\bar{r}_i$.

The orthogonal transformation A_{ij} represents the transformation from the $x_i y_i z_i$ coordinate frame to the $x_j y_j z_j$ frame. Thus:

$$\bar{x}_j = A_{ij}\bar{x}_i \tag{1}$$

The inverse transformation (from $x_j y_j z_j$ to $x_i y_i z_i$) may be indicated by A_{ji} or, equivalently, A_{ij}^T (the transpose of A_{ij}). The vector $\bar{r}$ is understood to have scalar components r_x, r_y, and r_z:

$$\bar{r} = [r_x \; r_y \; r_z]^T \tag{2}$$

while $\bar{r}_i$ has components:

$$\bar{r}_i = [r_{xi} \; r_{yi} \; r_{zi}]^T \tag{3}$$

REFERENCE COORDINATE FRAMES: (FIG 1)

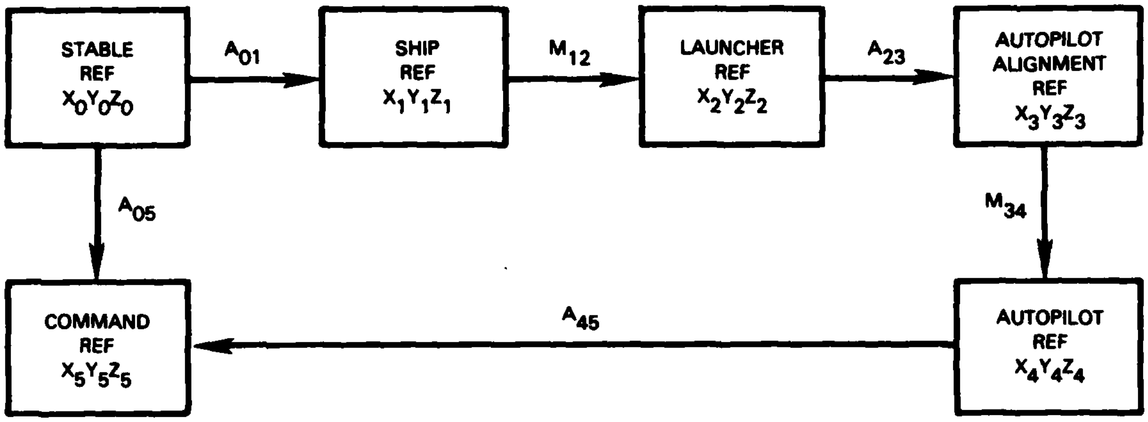

Figure 1. Reference Coordinate Frames

STABLE REFERENCE ($x_0y_0z_0$)

x_0 and y_0 lie in the local horizontal plane with positive x_0 true north and positive y_0 east (Fig 2). Positive z_0 is the local vertical-downward. We assume $x_0y_0z_0$ is an inertial reference frame.

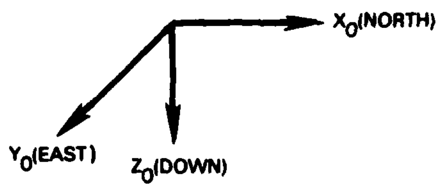

Figure 2. Stable Reference Frame

SHIP REFERENCE ($x_1y_1z_1$)

The ship reference axes (Fig 3) are the body axes of the ship used as the reference for measurement of the roll (ϕ), pitch (θ), and yaw (ψ) angles of the ship. These axes correspond, for example, to the axes of the MK 19 Gyro-compass or AN/WSN-5 Inertial System. The axes are fixed in attitude relative to the ship and rotate with the ship relative to the stable reference $(x_0 y_0 z_0)$. x_1 is the roll axis of the ship with positive x_1 forward, y_1 is the pitch axis, positive to starboard, and z_1 is the yaw axis with positive downward.

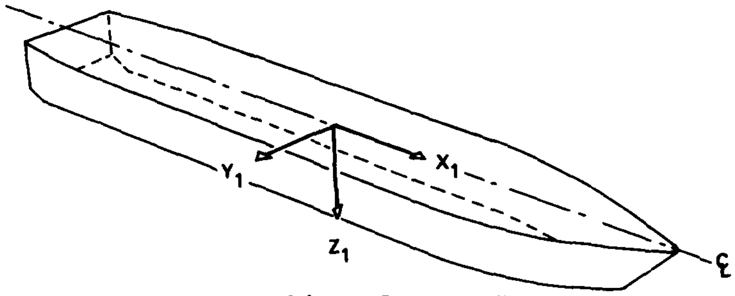

Figure 3. Ship Reference Frame

The attitude of the ship reference $(x_1 y_1 z_1)$ relative to the stable reference $(x_0 y_0 z_0)$ is a consequence of the ordered Euler angle rotations:

(1) ψ about the z-axis (yaw or heading)

(2) θ about the y-axis (pitch)

(3) ϕ about the x-axis (roll)

The orthogonal transformation from the stable reference $(x_0 y_0 z_0)$ to the ship reference $(x_1 y_1 z_1)$ is:

$$A_{01} = \begin{bmatrix} c\psi\ c\theta & s\psi\ c\theta & -s\theta \\ c\psi s\theta s\phi - s\psi c\phi & s\psi s\theta s\phi + c\psi c\phi & c\theta s\phi \\ c\psi s\theta c\phi + s\psi s\phi & s\psi s\theta c\phi - c\psi s\phi & c\theta c\phi \end{bmatrix} \tag{4}$$

where c and s denote the cosine and sine functions.

The ship reference frame, as defined above, corresponds to the reference frame conventionally used for attitude reference in virtually all modern technical publications, ie, positive roll and yaw motions are to starboard and positive pitch corresponds to a bow-up attitude. OP 1700 (Standard Fire

Control Symbols) does not conform to this convention, but instead defines positive roll motion to port and positive pitch as bow-down attitude. This report will continue to use the conventional definitions. The modifications required to use the OP 1700 definitions are presented in the Appendix.

LAUNCHER REFERENCE ($x_2y_2z_2$)

The vertical launching system, when installed, is nominally aligned with the ship reference axes ($x_1y_1z_1$). In fact, however, these axes are generally not used directly because of structural obstructions. Instead, the launching system is aligned with respect to some launcher reference ($x_2y_2z_2$) which was, in turn, previously aligned with respect to the ship reference ($x_1y_1z_1$). The transformation M_{12} represents the misalignment between the ship reference ($x_1y_1z_1$) and the launcher reference ($x_2y_2z_2$) and is a function of static and dynamic alignment errors due to errors in initial alignment and subsequent structural flexure.

AUTOPILOT ALIGNMENT REFERENCE ($x_3y_3z_3$)

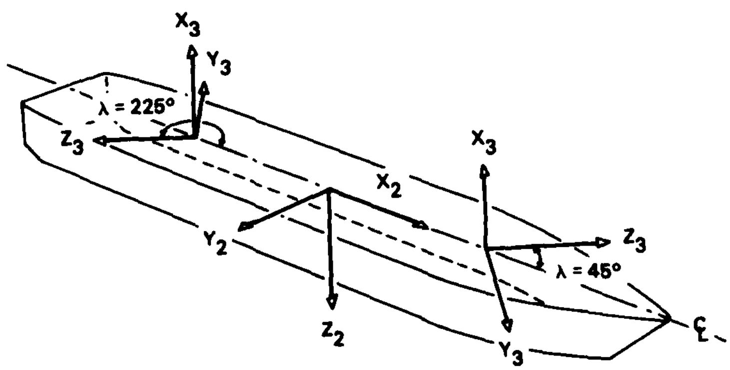

Figure 4. Autopilot Alignment Reference

The autopilot alignment reference axes ($x_3y_3z_3$) (Fig 4) represent the nominal (or intended) axes of the autopilot, that is, the autopilot axes when the missile, in its canister and cell, has been aligned, without error, with respect to the launcher reference ($x_2y_2z_2$). x_3 is the roll axis of the autopilot, y_3 the pitch axis, and z_3 the yaw axis. Positive x_3 is forward in the missile and positive y_3 is to starboard.

The configuration of the vertical launch system is such that, with the missile and its canister loaded into one of the cells, the positive x_3 axis is in the direction of negative z_2 and positive z_3 is either 45° to port of positive x_2 (we say $\lambda = 45°$) or 45° to starboard of negative x_2 (we say $\lambda = 225°$). Positive y_3 completes the right-handed triad.

The attitude of the autopilot alignment reference ($x_3y_3z_3$) relative to the launcher reference ($x_2y_2z_2$) is a consequence of the ordered Euler angle rotations:

(1) 90° about the y axis

(2) λ (45° or 225°) about the x axis

The orthogonal transformation from the launcher reference ($x_2y_2z_2$) to the autopilot alignment reference ($x_3y_3z_3$) is:

$$A_{23} = \begin{bmatrix} 0 & 0 & -1 \\ s\lambda & c\lambda & 0 \\ c\lambda & -s\lambda & 0 \end{bmatrix} \tag{5}$$

AUTOPILOT REFERENCE ($x_4y_4z_4$)

The autopilot reference axes ($x_4y_4z_4$) represent the actual reference axes of the autopilot (x_4-roll, y_4-pitch, z_4-yaw) with the missile and its canister loaded into a cell. The transformation M_{34} then represents the misalignment error of the autopilot axes ($x_4y_4z_4$) relative to the autopilot alignment reference ($x_3y_3z_3$).

COMMAND REFERENCE ($x_5y_5z_5$)

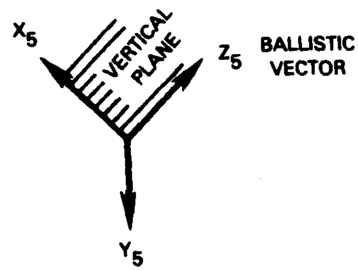

Figure 5. Command Reference

The command reference ($x_5y_5z_5$) is used as an attitude control reference by the autopilot (Fig 5). The positive z_5 axis is in the direction of the ballistic vector. Positive x_5 and positive z_5 lie in a vertical plane with positive x_5 upwards. y_5 completes the right-handed triad.

The ballistic vector is defined by an azimuth angle A measured in the horizontal plane from true north and an elevation angle E, the angle above the horizontal plane, measured in the vertical plane. Then the attitude of the command reference ($x_5y_5z_5$) relative to the stable reference ($x_0y_0z_0$) is a consequence of the following ordered Euler angle rotations:

(1) 90° about y (reorientation of axes)
(2) -A about x (azimuth)
(3) E about y (elevation)

The orthogonal transformation from the stable reference ($x_0y_0z_0$) to the command reference ($x_5y_5z_5$) is:

$$A_{05} = \begin{bmatrix} -cAsE & -sAsE & -cE \\ -sA & cA & 0 \\ cAcE & sAcE & -sE \end{bmatrix} \tag{6}$$

ATTITUDE OF COMMAND REFERENCE ($X_5Y_5Z_5$) RELATIVE TO AUTOPILOT REFERENCE ($x_4y_4z_4$)

The orthogonal tranformation A_{45} (Fig 1) describes the attitude of the command reference ($x_5y_5z_5$) relative to the autopilot reference ($x_4y_4z_4$). A_{45} is computed as the product of known matrices:

$$A_{45} = A_{05} A_{01}^T M_{12}^T A_{23}^T M_{34}^T \tag{7}$$

A_{01} and A_{05} are computed as indicated by Eq (4) and (6). A_{23} is computed as a function of λ as indicated by Eq (5). The parameter λ is a constant for a particular cell and will be provided (or some equivalent parameter) by the Launcher Control Unit (LCU) following cell selection.

The source of the misalignment transformations, M_{12} and M_{34}, is presently not known, pending results of current studies of system misalignment errors. These may be provided, in full or in part, by the LCU, may be computed in the fire control computer from externally provided data, or may be produced by some external subsystem dedicated to misalignment correction. In any event, the tranformations will be provided to the fire control computer for autopilot initialization.

PREDICTED ATTITUDE OF COMMAND REFERENCE $(x_5y_5z_5)$ RELATIVE TO AUTOPILOT REFERENCE $(x_4y_4z_4)$

Let the vector $\bar{\alpha}$ denote the angular velocity of the autopilot reference $(x_4y_4z_4)$ relative to the stable reference $(x_0y_0z_0)$ and let $\bar{\beta}$ represent the angular velocity of the command reference axes $(x_5y_5z_5)$ relative to the stable reference $(x_0y_0z_0)$.

The time derivative of the orthogonal transformation A_{45} is given by:

$$\dot{A}_{45} = A_{45}\Omega \tag{8}$$

where Ω is the skew-symmetric matrix:

$$\Omega = \begin{bmatrix} 0 & (\beta_{z4} - \alpha_{z4}) & (\alpha_{y4} - \beta_{y4}) \\ (\alpha_{z4} - \beta_{z4}) & 0 & (\beta_{x4} - \alpha_{x4}) \\ (\beta_{y4} - \alpha_{y4}) & (\alpha_{x4} - \beta_{x4}) & 0 \end{bmatrix} \tag{9}$$

The angular velocity vector $\bar{\alpha}$, in the absence of dynamic flexure, is identical to the ship's angular velocity vector $\bar{\omega}$, which can be computed from ship's roll, pitch, and yaw rates ($\dot{\phi}$, $\dot{\theta}$, $\dot{\psi}$, respectively). Then, assuming no dynamic flexure, we compute $\bar{\alpha}$ in ship reference $(x_1y_1z_1)$ coordinates:

$$\begin{aligned} \alpha_{x1} &= -\dot{\psi}s\theta + \dot{\phi} \\ \alpha_{y1} &= \dot{\psi}c\theta s\phi + \dot{\theta}c\phi \\ \alpha_{z1} &= \dot{\psi}c\theta c\phi - \dot{\theta}s\phi \end{aligned} \tag{10}$$

The angular velocity $\bar{\beta}$ must be computed from the dynamics of the command reference frame $(x_5y_5z_5)$. Specifically, the azimuth and elevation rates, $\dot{A}$ and $\dot{E}$, respectively, must be computed. Then $\bar{\beta}$, in stable reference coordinates $(x_0y_0z_0)$, is:

$$\beta_{x0} = -\dot{E}sA$$
$$\beta_{y0} = \dot{E}cA$$
$$\beta_{z0} = -\dot{A} \tag{11}$$

The differential equation [Eq (8)] provides the basis for prediction or update of the transformation A_{45}. If we wish to predict from time t to t + T, we can expand A_{45} in a Taylor series and use the first two terms as a first-order approximation:

$$A_{45}(t + T) \approx A_{45}(t) + T\dot{A}_{45}(t) \tag{12}$$

Other, more sophisticated, approximations are common in the literature and can be provided, if required.

LEAST SQUARES ORTHOGONALIZATION OF NONORTHOGONAL MATRICES

Because of numerical errors in digital computations and slightly nonorthogonal reference axes, algorithms for the matrix equations described herein generally produce slightly nonorthogonal matrices. The nonorthogonality can, in some instances, produce adverse effects. Thus it is common in such computations to include some subroutine to orthogonalize the computed transformations. A first-order, least-squares algorithm for orthogonalization is described below.

Let A represent a computed transformation matrix which is "near-orthogonal." The least squares algorithm:

$$\tilde{A} = A\,(A^T A)^{-1/2} \tag{13}$$

provides an orthogonal transformation $\tilde{A}$ which is nearest to A in a least squares sense. That is, the algorithm minimizes the error measure e:

$$e = TR[(A-\tilde{A})^T(A-\tilde{A})] \tag{14}$$

(*where TR indicates* the matrix trace function) subject to the constraint that $\tilde{A}$ be orthogonal:

$$\tilde{A}\tilde{A}^T - I = 0 \tag{15}$$

Equation (13) is computationally difficult because the square root function requires computation of eigenvalues. In practice, a first-order approximation is usually sufficient:

$$\tilde{A} = A + 1/2\,A\,(I - A^T A) \tag{16}$$

Other, more sophisticated, orthogonalization algorithms are common in the literature.

VELOCITY OF THE LAUNCH POINT REFERENCE RELATIVE TO THE MOVING AIR MASS

We define the following vectors (see Fig 6):

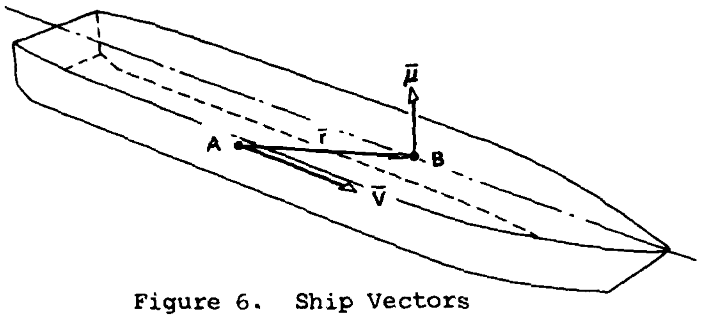

Figure 6. Ship Vectors

(1) $\bar{v}$ = ship's inertial velocity measured at some reference point A in the ship.

(2) $\bar{\mu}$ = inertial velocity of the launch point reference B.

(3) $\bar{r}$ = displacement vector from point A (ship's velocity reference) to point B (launch point reference).

(4) $\bar{\omega}$ = angular velocity of the ship relative to the stable reference $(x_0y_0z_0)$.

(5) $\bar{w}$ = inertial wind velocity.

Since we have assumed that the stable reference $(x_0y_0z_0)$ is an inertial reference, all velocities and angular rates are relative to that reference.

Autopilot initialization requires the velocity of the launch point reference (B) relative to the moving air mass in autopilot reference coordinates $(x_4y_4z_4)$, that is, the vector quantity $\bar{\mu}_4 - \bar{w}_4$.

The vectors $\bar{v}$ (ship's velocity) and $\bar{w}$ (wind velocity) are measurements in stable reference coordinates $(x_0y_0z_0)$ provided to the fire control system. The vector $\bar{r}$ is provided by the LCU when a particular cell has been selected. We assume that $\bar{r}$ is provided in ship reference coordinates $(x_1y_1z_1)$.

The angular velocity vector $\bar{\omega}$ may be computed in ship reference coordinates $(x_1 y_1 z_1)$ from ship roll, pitch, and yaw rates by means of Eq (10):

$$\begin{aligned} \omega_{x1} &= -\dot{\psi} s\theta + \dot{\phi} \\ \omega_{y1} &= \dot{\psi} c\theta s\phi + \dot{\theta} c\phi \\ \omega_{z1} &= \dot{\psi} c\theta c\phi - \dot{\theta} s\phi \end{aligned} \tag{17}$$

Then:

$$\begin{aligned} \bar{\mu}_4 - \bar{w}_4 &= \bar{v}_4 + (\bar{\omega}_4 \times \bar{r}_4) - \bar{w}_4 \\ &= M_{34}\, A_{23}\, M_{12} [A_{01} (\bar{v}_0 - \bar{w}_0) + (\bar{\omega}_1 \times \bar{r}_1)] \end{aligned} \tag{18}$$

PREDICTED VELOCITY OF THE LAUNCH POINT REFERENCE RELATIVE TO THE MOVING AIR MASS

A simple linear extrapolation algorithm can be used to provide a first-order estimate of launch point reference velocity in the moving air mass at some future time.

To simplify notation, let:

$$\bar{x} = \bar{\mu}_4 - \bar{w}_4 \tag{19}$$

Then, a first-order estimate of $\bar{x}$ at time $t + T_1$ can be determined from computed values of $\bar{x}$ at times t and $t - T_2$:

$$\bar{x}\,(t + T_1) \approx \bar{x}(t) + T_1\left[\frac{\bar{x}(t) - \bar{x}(t-T_2)}{T_2}\right] \tag{20}$$

Other, more complex nonlinear extrapolation algorithms may be used, if this simple linear extrapolation is inadequate.

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APPENDIX

MODIFIED EQUATIONS FOR OP 1700 DEFINITIONS

OP 1700 (Standard Fire Control Symbols) defines own-ship yaw motion as positive to starboard, roll motion as positive to port, and positive pitch as bow-down attitude. Using these definitions, Fig A-1 illustrates the ship reference frame $(x_1y_1z_1)$. The attitude of the ship reference $(x_1y_1z_1)$ relative to the stable reference $(x_0y_0z_0)$ is a consequence of the following ordered Euler angle rotations:

(1) 180° about the z-axis (reorient axes)

(2) ψ about the z-axis (yaw)

(3) θ about the y-axis (pitch)

(4) ϕ about the x-axis (roll)

Then the orthogonal transformation from the stable reference $(x_0y_0z_0)$ to the ship reference is:

$$A_{01} = \begin{bmatrix} -c\psi c\theta & -s\psi c\theta & -s\theta \\ -c\psi s\theta s\phi + s\psi c\phi & -s\psi s\theta s\phi - c\psi c\phi & +c\theta s\phi \\ -c\psi s\theta c\phi - s\psi s\phi & -s\psi s\theta c\phi + c\psi s\phi & +c\theta c\phi \end{bmatrix} \tag{A-1}$$

The launcher reference frame $(x_2y_2z_2)$ and ship reference frame $(x_1y_1z_1)$ are assumed to be nominally aligned. Thus a change in orientation of the ship reference $(x_1y_1z_1)$ requires a corresponding reorientation of the launcher reference $(x_2y_2z_2)$. Therefore the transformation A_{23} will change, since the launcher reference $(x_2y_2z_2)$ is reoriented without a corresponding reorientation of the autopilot alignment reference $(x_3y_3z_3)$.

The transformation from the launcher reference $(x_2y_2z_2)$ to the autopilot alignment reference is a consequence of the following ordered Euler angle rotations (see Fig A-2):

(1) 180° about the z-axis

(2) 90° about the y-axis

(3) λ (45° or 225°) about the x-axis

Then the transformation A_{23} is given by:

$$A_{23} = \begin{bmatrix} 0 & 0 & -1 \\ -s\lambda & -c\lambda & 0 \\ -c\lambda & s\lambda & 0 \end{bmatrix} \qquad \text{(A-2)}$$

No other modifications to the equations in the body of this report are necessary, when the OP 1700 definitions of own-ship roll, pitch, and yaw are used.

To summarize: When using the OP 1700 definitions in place of the conventional definitions, replace Fig 3 and 4 with Fig A-1 and A-2, respectively, and replace Eq (4) and (5) with Eq (A-1) and (A-2), respectively.

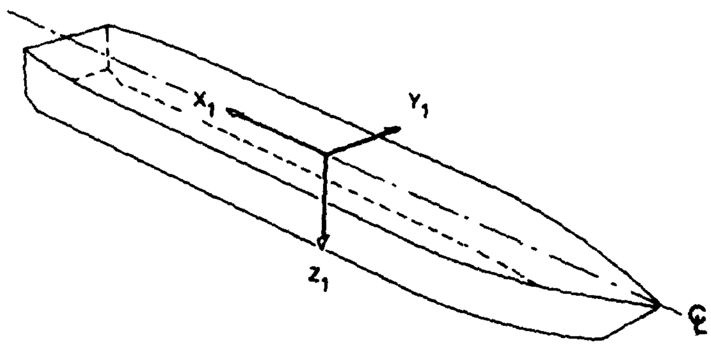

Figure A-1. Ship Reference Frame (OP 1700 Standard)

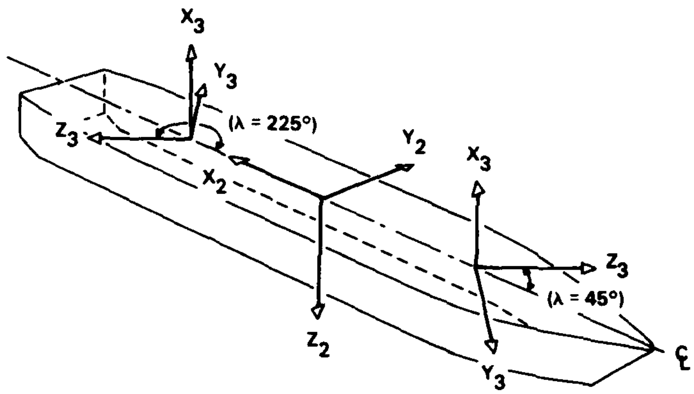

Figure A-2. Autopilot Alignment Reference ($X_3Y_3Z_3$) and Launcher Reference Frame ($X_2Y_2Z_2$) (OP 1700 Standard)